

Is there a Bigger Fix in the Multiverse?

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The dynamics of baby universes which are branched off in pairs from a large universe and whose respective members are entangled to each other has been studied in quantum gravity and string theory. It is shown that the probability measure for such pairs essentially keeps its Planckian form when one of their two members is trapped by a Lorentzian tunnel and travels through it into another large universe, so that the two baby universes of the pairs preserve their mutual entanglement even when they are going to be finally branched in distinct large universes. The conclusion is thus drawn that big fixing the fundamental constants in our universe actually big fixes such constants in the set of all single universes of the multiverse.

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One of the reasons why the old ideas of quantum cosmology and Euclidean quantum gravity have been retaken with renewed interest in recent years is the growing recognition that the landscape of the stringy vacua can be too big to allow that reasonable selection criteria for particle phenomenology and initial conditions for the universe can be successfully worked out within string theory itself [1]. Among such ideas there was what Coleman dubbed the Big Fix [2], according to which the minima of the Euclidean action provided the most probable values for the fundamental constant, including a vanishing value for the cosmological constant. This result corresponded to a wave function for the universe which was expressed in terms of the exponential of minus the Euclidean action. That was rather confusing and counter-intuitive [3] because it predicted higher probability for the largest universes and, more importantly, because it is now seen to contradict possible interpretations of the current accelerating expansion of the universe which can only be compatible with small but still nonzero value for the cosmological constant or the energy density of dark energy [4]. At the end of the day, quantum cosmology expressed in terms of the Wheeler-DeWitt equation also allows for the possibility that the wave function of the universe can be described as the exponential of plus the Euclidean action [5], so opening the room for conclusions other than attributing a small probability for a large value of the cosmological constant.

There is still another reason that can contribute to switch on an additional interest in the Coleman Euclidean program when it is conveniently supplemented with some extra ingredients. It is that baby universes - that is the Lorentzian version of Euclidean wormholes [6] which were explicitly used by Coleman [2] in order to establish the big fix mechanism leading to the fixing of the fundamental physical constant - need not be created individually as it was initially assumed in Coleman dynamics, but they can also be branched off from our universe in correlated baby-universe pairs [7] which should only be described by means of a density matrix [8] rather than a wave function. In this case, there will al-

ways be a non vanishing contribution from a mixed state density matrix to describe the quantum state of the Euclidean wormholes. It was shown more than fifteen years ago that in such a case, the probability measure derived by using quantum gravity [9] and string theory [10] arguments appears to be free from all confusing situations and counter-intuitiveness shown by the Coleman case and actually predicts the result required by the current accelerating evolution of the universe that the most probable value of the cosmological constant might be small but always nonzero, and hence the associated big fix appears to be quite more reliable.

I will first very briefly review here the results derived from quantum gravity and string theory respectively at Refs. [9] and [10], in a rather qualitative fashion. In the dilute Euclidean wormhole approximation [6], the effects produced by a single wormhole on ordinary matter fields in the asymptotic regions can be expressed in the path integral for the expectation value of any given observable O by inserting a factor which, if the Euclidean wormhole quantum state is given by a wave function, does not depend on any energy-spectrum characteristic. According to the formalism put forward by Klebanov, Susskind and Banks [11] we thus find that, after summing over the number of Euclidean wormholes (so exponentiating the action integral) and making then the bi-local action local (in this way entering the Coleman α -parameters), one gets the Coleman simple exponential probability law [12] that leads to the uncertainties and shortcomings pointed out before. No further summation should be here performed as the resulting expression does not depend on any Euclidean wormhole spectrum or other discrete indices.

However, if the baby universes are branched off and in in pairs, after performing the above two steps which had to be also performed in the individual baby universe case (that is, summing over the number of wormholes and making local the resulting exponent), then one has [9,10] to also sum over the energy spectral index $k = m - n \geq 1$ on which the expression resulting from the two first steps depends. The dynamics is in that case described in terms

of a Planckian-like law [9,10] which gives a maximal probability of order unity not when the cosmological constant vanishes but whenever it gets a nonzero value which can be done so small as the Coleman's alpha parameter allows it. The fixing of the other constant of physics will similarly be determined by other minima of the Euclidean action governed by that Planckian-like probability law. It is worth noticing moreover that, if we normalize the probability to a maximal value unity, then smaller universes would be more probable than larger ones, and hence inflation is predicted to be favored. The quantum content represented by such a law corresponds to a quantization of gravity which is over and above that is contained in the usual formulation of quantum cosmology. It should necessarily be expressing the only distinctive feature introduced in the derivation of the law that the two baby universes of the pairs are correlated to each other. Nucleating a pair of baby universes the two at once looks quite the same as the emission of two entangled photons by a nonlinear optical process in EPR experiments. The main hypothesis of the present paper is to interpret the Planckian shape of the quantum-gravity probability law as a direct consequence from an the entanglement between the two baby universes in each branched off pair, in such a way that if one determines the state of one of the two baby universes of one such pairs the state of the other baby universe of the same pair is automatically determined as well, even in the case that we do not perform any measurement on the latter baby universe.

In what follows we shall consider a framework in which an entangled baby universe pair is branched off from a parent universe, denoted Universe I (see Fig. 1), of which just one of the baby universes is trapped by a Lorentzian space-time tunnel through which it travels into another parent universe, denoted Universe II (see Fig. 1), at which it is finally branched in, leaving the tunnel, while the non trapped baby universe of the original pair branches in back in parent universe I. Any baby universe that enters a tunnel should be expected to couple with the tunnel as a whole from one mouth to the other, and the baby universe-tunnel coupling would only depend on the kind of tunneling - whether it is a wormhole, a ringhole or a Klein-bottle hole - so as on the relative velocity of its two mouths and the physical parameters and topology that define its throat. When one allows the two wormhole mouths to move relative to each other with a nonzero velocity, then the trapped baby universe would be expected to travel in time.

One expects that any of these coupling and mouth vibrating processes will generally take place without any relevant change in the mutual entanglement between the two baby universes. The physical reason for that is that the two compact, closed constructs branched off from the parent universe do not originally evolve in any background Lorentzian space-time, so that their mutual entanglement cannot be defined to depend on any background Lorentzian space-time. Thus, the space-time placement of the baby universe along the Lorentzian

wormhole tunneling cannot be involved at the two baby universe mutual entanglement. Therefore, the unique blueprint that any traveling of one baby universe along the tunnel would leave in the probability measure defined in terms of a path integral would just be in the form of a given overall coupling constant ξ_j (if it is baby universe j which couples to the Lorentzian tunnel, see Fig. 1) entering the action integral, and this modification implies no further summation or integration (tracing off) requirement for deriving the dynamics, and hence the resulting probability expression should keep its Planckian shape.

In order to work out the precise form of the dynamics and derive from it the physical implications for the situation depicted in Fig. 1, we shall proceed as follows. In the dilute Euclidean wormhole approximation [6,12], the effects produced by Euclidean wormholes on ordinary matter fields on the asymptotic regions of Universe I can be expressed in the path integral for the expectation value of a given observable O by a double insertion: a factor which, if the Euclidean wormhole quantum state is given by a density matrix, is $-\frac{1}{2}C_{ij}\beta_i\beta_j\epsilon_{mn}^{-1}$, and a factor $\gamma_j = \xi_j^{-1}$ inversely depending on the overall coupling of the Lorentzian baby universe j to the Lorentzian wormhole (see Fig. 1). We then have [9]

$$\langle O \rangle = \int dg O e^{-I(g,\lambda)} \left(\frac{1}{2} \sum_{i,j} C_{ij} \gamma_j(y) \beta_i(x) \beta_j(y) \epsilon_{mn}^{-1} \right), \quad (1)$$

where $C_{ij} \propto e^{-S_w} = D_{ij}^{-1}$, with S_w the Euclidean action for the Euclidean wormhole, $\epsilon_{mn} = E_m^{(f)} - E_n^{(g)}$, the E 's being the energy levels for the matter fields (f) and the gravitational field (g) harmonic oscillators, respectively, and ϵ_{mn}^{-1} the relative probability for the given state Ψ_{mn} [8]. λ collectively denotes parameters such as coupling constants, particle masses, the cosmological constant, etc.; $\beta_i = \frac{1}{\sqrt{2}} \int d^4x \sqrt{g(x)} V_i(x)$, with V_i denoting the vertex operator and index i labeling the elements of the basis for the local field operators at the point x on the large region. Indices i, j are independent of the quantum numbers n, m which label the off shell-energy spectrum. All dependence of the path integral on that spectrum and on the coupling of the baby universe to a Lorentzian tunnel are incorporated through the relative probability factor ϵ_{mn}^{-1} and the factor γ_j , respectively, because the respective unique invariant theories for asymptotically flat Euclidean wormholes and asymptotically flat space-time tunnels must satisfy the boundary requirement that all possible kinds of Euclidean wormholes and Euclidean wormhole states, on the one hand, and Lorentzian tunnels and its possible classical or even quantum states, on the other hand, ought all to have equal asymptotic behavior, respectively, thus rendering the insertion amplitude to join their ends onto the asymptotic regions independent of the Euclidean wormhole spectrum and the baby universe-Lorentzian wormhole coupling. The dependence on the eigenenergies ϵ_{mn} arises because the Euclidean wormhole is off-shell in the case that the baby

universes are nucleated in pairs. If such universes are Planck-sized, then we have the most probable case where $\epsilon_{m,n} = m - n$. Now, following Coleman [2], we first sum over any number of Euclidean wormholes, and then make local the resulting exponent, to get

$$\int dg O e^{-I(g,\lambda)} \int \prod_p d\alpha_p e^{-\frac{1}{2}(m-n)D_{ij}\xi_j\alpha_i\alpha_j} e^{-\beta_i\alpha_i}, \quad (2)$$

through which the position-independent parameters α 's enter the formalism.

Now, unlike the case considered by Coleman [2,12], we should next sum [9,10] over all possible Euclidean wormhole states. Avoiding the over counting that comes from summing over n and m independently, so as restricting to indices such that $m > n$, in order to ensure a converging path integral, and taking the lower limit in the sum to be unity, in order to keep the off-shell character even classically, we finally obtain

$$\langle O \rangle \propto \int d\alpha P(\alpha) Z(\alpha) \langle O \rangle_{\lambda+\alpha}, \quad (3)$$

where the third exponent of the previous expression has been inserted in the path integral of the parent universe I, and

$$P(\alpha) = \frac{1}{e^{D_{ij}\xi_j\alpha_i\alpha_j} - 1}, \quad Z(\alpha) = \int dg e^{-I(g,\lambda+\alpha)}. \quad (4)$$

These results can be rather straightforwardly confirmed in the two-dimensional realm of string theory. In fact, one can study Euclidean multi-wormhole configurations in Polyakov stringy theory by looking at the Euclidean wormholes as [10,13] the handles on a Riemann surface of genus ρ , with ρ the number of handles or Euclidean wormholes in the configuration in the case depicted in Fig. 1. Starting thus with the Green function that describes the effects of the handles on first order tachyonic amplitudes taken as the path integral over all space-time coordinates on the Riemann surface, and by following the calculation performed in Ref. [10], supplemented by the arguments on the space-time independence of the coupling constant between the baby universes and the Lorentzian tunnels, stated before, we finally obtain the same result as from quantum gravity, provided we express the Green function in terms of the handle quantum state on the circle using the Fourier transform of the delta function [13] (ensuring that, after cutting the handles, points on a resulting circle are identified with points on the other resulting circle) for the zero mode, and then expanding the delta function for the nonzero modes in terms of the complete set of the orthonormal harmonic-oscillator eigenstates that are the solutions of the string analogous of the Wheeler-DeWitt equation [13]. It can be shown that this procedure leads to a bi-local effective action given by

$$- \int d\sigma_1 \int d\sigma_2 \sum \int d^4 K \kappa \xi_2 V_p(\sigma_1) V_p(\sigma_2), \quad (5)$$

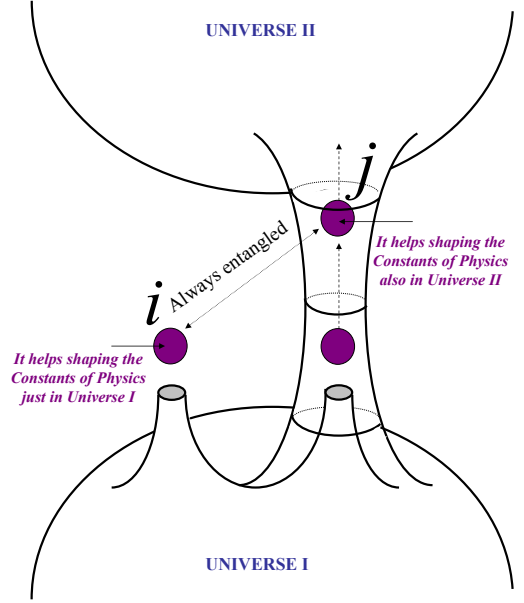


FIG. 1: Pictorial representation of the nucleation of an entangled baby universe pair in the context of the multiverse, with one of the baby universes branching off freely from the parent universe, and the other being trapped by a Lorentzian tunnel (a wormhole, a ringhole or a klein-bottle hole) along which it may travel to the other universe. After a short time the baby universe trapped in the Lorentzian tunnel arrives at and is branched in the universe other than the one where it was originally branched off, time traveling or not. The important result is that the entanglement between the two baby universes is expected to be preserved in both cases, so that determining the values of the physical constants of Universe I according to the improved Coleman's big fix mechanism considered here immediately determines the values of those constants of Universe II.

where the V_p 's are the handle vertex operators for the fields on the two circles, $\kappa = \left(K^2 + \sum |n| m_n^{(i)} - 2\right)^{-1}$, which results from integrating over the fields, with K the momentum for the zero mode $n = 0$, and m_n labeling the excited states of the resulting Wheeler-DeWitt wave function [13], $e^{-\frac{1}{2}n(Y_n^{(i)})^2} H_{m_n^{(i)}}(\sqrt{n}Y_n^{(i)}) e^{iKx_0}$, with $i = 1, 2$, $Y_n^{(1)} = \frac{1}{2}(x_n + x_{-n})$, $Y_n^{(2)} = \frac{1}{2i}(x_n - x_{-n})$, and the coupling constant ξ_2 means that we have chosen position 2 to be occupied by the baby universe which enters the Lorentzian tunnel. We then first convert action (5) into a local quantity by inserting the Coleman parameters,

summing then over all m_n , to finally have [10]

$$Z(\alpha) \prod_q \left(e^{\frac{1}{2} n^2 \xi_2 \alpha_q^2} - 1 \right)^{-1}, \quad (6)$$

which, when one identifies $D_{ij} \equiv n^2$ and $2 \equiv j$, and takes into account that, out from all possible combinations of indices i, j , only the diagonal combination $i = j = q$ is allowed by quantum requirements [10], becomes fully equivalent to Eqns. (3) and (4) which was obtained in the context of four-dimensional quantum gravity.

If the different single universes of the multiverse are mutually interconnected to each other by means of space-time tunnels, such as it must be expected to happen, there then will always be a nonzero contribution to the path integral describing the state of the multiverse from branched off baby universe pairs so that there always be individual baby universes traveling through Lorentzian tunnels which are entangled to their partner in the parent universe in which they were both originally branched off as an entangled pair, so that all universes in the multiverse are connected to each other in such a way that if you determine by means of measurements the values the fundamental constants in a given universe you are at the same time determining the value of such constants in all other universes in the multiverse. We obtain thus a bigger fix for the fundamental constants both in our universe and in the rest of universes of the considered multiverse.

On the other hand, from inspection of Eqns (3) and (6) we can deduce:

- Entanglement between two baby universes is always preserved unless for the extreme case in which one of them very strongly couples to a space-time tunnel.
- If such a coupling is weak enough then the average number of existing baby universe pairs should tend to be larger so that the whole set tends toward its classical limit.

- If the coupling is strong then the average number of existing baby universe pairs should tend to be small and separates from the classical behavior.
- Finally, if the two Lorentzian tunnel mouths move with velocity v relative to one another, then the above coupling would be expected to depend on v in such a way that the larger v the larger ξ . One thus might tentatively assume $\xi \propto \frac{2\xi^{(o)}}{2-v}$. It would then follow that time traveling of one of the baby universes of a pair can never completely destroy mutual entanglement between the two baby universes.

The main conclusion coming from the above calculations and the consequences drawn from them is that when one settles the values of the fundamental constants in a given single universe, such constants and laws should also be settled down once and for all in the rest of single universes of the multiverse. It follows that, in spite of the classical mutual independence of the single universes of one another, if the existence of some classical tunneling mechanism connecting the distinct single universes exhaustively among them is allowed, and the nucleation of mutually entangled baby universe pairs is permitted, with one of these baby universes being trapped by the tunneling network mechanism, then it is possible to conceive lab experiments or physical observations on Earth which will check the physical existence of other single universes in the multiverse.

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